ANALYTICAL SOLUTION TO THE POLYNOMIAL DIOPHATINE EQUATION: DEVELOPMENT AND APPLICATION TO GENERALISED PREDICTIVE CONTROL ANALYSIS

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ABSTRACT

The techniques of predictive control are based on building a prediction about the future plant behaviour. One way to obtain such a prediction requires solving a Polynomial Diophantine Equation (PDE). There are different methods to solve such an equation, which are good for design purposes but present some restrictions when studying the behaviour of solutions and the performance of the methods in which they are applied.
In this article an analytical solution for the PDE is developed, which constitutes a new analysis tool for predictive methods. Also, first results are briefly described on the application of this solution to the analysis of the Generalised Predictive Control method.

Keywords: diophantine equation, predictive control

INTRODUCTION

Model Based Predictive Control (MBPC) is a family of discrete controller design methods that have been under development for more than twenty years.
These methods are based on the prediction at a discrete time k of the effect that future inputs would have in the plant behaviour at times $k+j$, $j=1,...,M$ (M is called the prediction horizon) and they require knowledge on the system model and its past behaviour.
Among the last developments of methods belonging to this family are: Generalised Predictive Control (GPC), (Clarke et al (2)), Constrained Receding Horizon Predictive Control (CRHPC) (Clarke and Scattolini (5)) and Stabilizing Input/Output Receding Horizon Control (SIORHC) (Mosca and Zhang (6)).
One of many forms to predict system behaviour is based on the solution of a polynomial diophantine equation (PDE), whose coefficients depend on the plant discrete transfer function and on index j. This equation must be solved for each value of $j = 1,..., M$.
Several forms to solve the PDE have been described, of which the recursive equation approach is given later (see (2) and Plarre and Rojas (12)). In general these methods are very useful for design purposes but present some restrictions to analyse the behaviour of solutions as index j is varied and, consequently, do not lend themselves to make some assessments on the method under study.
In this article an analytical solution to the PDE is developed. The idea is based on the recursive approach presented in (2), and the result allows to study the behaviour of the PDE solutions when j changes, giving rise to a new analysis tool on prediction methods. Also, first results are described on the application of this solution to the analysis of the GPC method (see (12)).

1 POLYNOMIAL DIOPHANTINE EQUATION

The PDE applied in predictive control takes the form:

$$R(q^{-1}) = E_j(q^{-1})\tilde{A}(q^{-1}) + q^{-j}F_j(q^{-1}) \tag{1}$$

this equation must be solved for E_j and F_j, where:

q^{-1}	is the unit delay operator
j	is the desired prediction time
$R(q^{-1})$	$= r_0 + r_1q^{-1} + \ldots + r_{nr}q^{-nr}$
$\tilde{A}(q^{-1})$	$= (1-q^{-1})A(q^{-1})S(q^{-1})$
	$= 1 + \tilde{a}_1q^{-1} + \ldots + \tilde{a}_{n+1}q^{-n-1}$ (2)
n	$= na + ns$
$A(q^{-1})$	$= 1 + a_1q^{-1} + \cdots a_{na}q^{-na}$ (3)
	is normally the plant denominator
$S(q^{-1})$	$= 1 + s_1q^{-1} + \ldots + s_{ns}q^{-ns}$
	By design $n \geq ns \geq nr$
	R/S is provided to deliver a filtered plant output version
$E_j(q^{-1})$	$= e_{j,0} + e_{j,1}q^{-1} + \ldots + e_{j,j-1}q^{-j+1}$ (4)
	is the first unknown polynomial
$F_j(q^{-1})$	$= f_{j,0} + f_{j,1}q^{-1} + \ldots + f_{j,n}q^{-n}$ (5)
	is the second unknown polynomial

Argument q^{-1} in polynomials is dropped for simplicity.

2 THE RECURSIVE SOLUTION APPROACH

As a starting point to develop the analytical solution to be proposed, the recursive solution is given first.
The recursive approach (see (2)) uses the solution to equation (1) for j to obtain the solution for $j+1$.

The recursive solution is given by:

$$E_{j+1} = E_j + e_{j+1,j}q^{-j} \tag{6}$$

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$$e_{j+1,j} = f_{j,0} / \tilde{A}(0) \quad (7)$$

$$f_{j+1,i} = f_{j,i+1} - \tilde{a}_{i+1} f_{j,0} \quad (8)$$

In order to start iterations, E_1 and F_1 are given below:

$$E_1 = R(0)/S(0) \quad (9)$$

$$F_1 = (R - \tilde{A}R(0)/S(0))q \quad (10)$$

This solution to the PDE requires fewer calculations than the solution by term comparison given in Aström and Wittenmark (11). However the approach does not allow to get solutions from a j greater than 1, without calculating the previous ones. This may be so in GPC when, for example, it is necessary to predict from an initial time greater than the time delay of the plant under control. Also, it is complex to study the behaviour of solutions as functions of the prediction horizon.

3 ANALYTICAL SOLUTION PROPOSED

To begin, (8) is written for $j-1$:

$$f_{j,i} = f_{j-1,i+1} - \tilde{a}_{i+1} f_{j-1,0} \quad (11)$$

Now Z transform is applied with respect to index j:

$$F_i(z) = z^{-1}F_{i+1}(z) + f_{0,i+1} - \tilde{a}_{i+1}z^{-1}F_0(z) - \tilde{a}_{i+1}f_{0,0} \quad (12)$$

where $F_i(z)$ is the Z transform of $f_{j,i}$. Notice that since index j takes values from $j=1$ onwards, the initial condition for $f_{j,i}$ is $f_{0,i}$.
Now, (12) is written in explicit form for each value of index i:

$$F_0(z) = z^{-1}F_1(z) + f_{0,1} - \tilde{a}_1 z^{-1}F_0(z) - \tilde{a}_1 f_{0,0}$$
$$F_1(z) = z^{-1}F_2(z) + f_{0,2} - \tilde{a}_2 z^{-1}F_0(z) - \tilde{a}_2 f_{0,0}$$
$$F_2(z) = z^{-1}F_3(z) + f_{0,3} - \tilde{a}_3 z^{-1}F_0(z) - \tilde{a}_3 f_{0,0}$$
$$\ldots$$
$$F_{n-1}(z) = z^{-1}F_n(z) + f_{0,n} - \tilde{a}_n z^{-1}F_0(z) - \tilde{a}_n f_{0,0}$$

$$F_n(z) = -\tilde{a}_{n+1}z^{-1}F_0(z) - \tilde{a}_{n+1}f_{0,0} \quad (13)$$

By replacing backwards, ($F_n(z)$ in $F_{n-1}(z)$, and the result in $F_{n-2}(z)$, etc.) an expression for $F_0(z)$ is found:

$$F_{n-1}(z) = -[\tilde{a}_{n+1}z^{-2} + \tilde{a}_n z^{-1}]F_0(z) - -[\tilde{a}_{n+1}z^{-1} + \tilde{a}_n]f_{0,0} + f_{0,n}$$

$$F_{n-2}(z) = -[\tilde{a}_{n+1}z^{-3} + \tilde{a}_n z^{-2} + \tilde{a}_{n-1}z^{-1}]F_0(z) - -[\tilde{a}_{n+1}z^{-1} + \tilde{a}_n]f_{0,0} + f_{0,n}z^{-1} + f_{0,n-1}$$

$$\ldots$$

$$F_i(z) = -[\tilde{a}_{n+1}z^{i-n-1} + \tilde{a}_n z^{i-n} + \cdots + \tilde{a}_{i+1}z^{-1}]F_0(z) - -[\tilde{a}_{n+1}z^{i-n} + \tilde{a}_n z^{i-n+1} + \cdots + \tilde{a}_{i+1}]f_{0,0} + +[f_{0,n}z^{i-n+1} + f_{0,n-1}z^{i-n+2} + \cdots + f_{0,i+1}] \quad (14)$$

$$F_0(z) = -[\tilde{a}_{n+1}z^{-n-1} + \tilde{a}_n z^{-n} + \cdots + \tilde{a}_1 z^{-1}]F_0(z) - [\tilde{a}_{n+1}z^{-n} + \tilde{a}_n z^{-n+1} + \cdots + \tilde{a}_1]f_{0,0} + +[f_{0,n}z^{-n+1} + f_{0,n-1}z^{-n+2} + \cdots + f_{0,1}] \quad (15)$$

By ordering terms in (15):

$$F_0(z) + [\tilde{a}_{n+1}z^{-n-1} + \tilde{a}_n z^{-n} + \ldots + \tilde{a}_1 z^{-1}]F_0(z) = -[\tilde{a}_{n+1}z^{-n} + \tilde{a}_n z^{-n+1} + \ldots + \tilde{a}_1]f_{0,0} + +[f_{0,n}z^{-n+1} + f_{0,n-1}z^{-n+2} + \ldots + f_{0,1}] \quad (16)$$

an equivalent expression for the left side is found:

$$\tilde{A}F_0(z) = -[\tilde{a}_{n+1}z^{-n} + \tilde{a}_n z^{-n+1} + \ldots + \tilde{a}_1]f_{0,0} + +[f_{0,n}z^{-n+1} + f_{0,n-1}z^{-n+2} + \ldots + f_{0,1}] \quad (17)$$

Now (17) may be multiplied by z^{-1}, $f_{0,0}$ may be added and subtracted from the term in the right, and by recognising polynomials, one gets:

$$z^{-1}\tilde{A}F_0(z) = -[\tilde{a}_{n+1}z^{-n-1} + \tilde{a}_n z^{-n} + \cdots + \tilde{a}_1 z^{-1} + 1]f_{0,0} + +[f_{0,n}z^{-n} + f_{0,n-1}z^{-n+1} + \cdots + f_{0,1}z^{-1} + f_{0,0}] \quad (18)$$

therefore

$$z^{-1}\tilde{A}F_0(z) = -\tilde{A}f_{0,0} + F_0$$

and finally,

$$F_0(z) = [F_0 / \tilde{A} - f_{0,0}]z \quad (19)$$

The value of F_0 may be found from (1) with $j=0$:

$$R = \tilde{A}E_0 + F_0 q^0 \quad (20)$$

Since $E_0=0$, we have $F_0=R$, and hence (19) yields

$$F_0(z) = [R/\tilde{A} - R(0)]z \quad (21)$$

From (4), the degree of E_j is j-1. From (6), only the last coefficient of E_j changes according to (7).
From (7), E_j is given by

$$E_j = 1 + f_{1,0} q^{-1} + \cdots + f_{j-1,0} q^{-j+1}$$

From (14), it is noted that the remaining components of polynomial F_j, $f_{j,i}$ (for $i=1,2,...$) are linear combinations of $f_{j,0}$, $f_{j-1,0}$, etc.

4 APPLICATION TO GPC ANALYSIS

The analytical solution to PDE obtained in 3 was applied to analyse the GPC method. In what follows little more than results are given, details may be found in (12).
Since polinomial E_j and components $f_{j,1}$, $f_{j,2}$, etc. of polinomial F_j can be obtained from $f_{j,0}$, the study of this last element is the fundamental basis for the analysis.

4.1 The G.P.C. method

Below the fundamentals of GPC, necessary for the developments to follow, are given. For details see (2) and Yamamoto, Omatsu and Kaneda (8).

The GPC method obtains the actuation to be applied at discrete time k by minimising a cost function that includes a weighted sum of future predicted errors and future input variations. This last term is necessary to avoid overshoots.

$$J(k) = E\left\{ \sum_{j=1}^{M} [\hat{y}(k+j\,|\,k) - w(k)]^2 + \sum_{j=1}^{M} \lambda[\Delta u(k+j-1)]^2 \right\} \quad (22)$$

where:

- $E\{\}$ is the mathematical expectation.
- $\hat{y}(k+j|k)$ is the predicted system output at time j from the information available at time k.
- $w(k)$ is the reference at time k. It is supposed to be constant for the whole prediction horizon.
- Δ $= 1 - q^{-1}$
- λ it establishes the relative importance between minimising future errors and keeping input bounded.
- M is the prediction horizon.

The model used to describe the plant is given by:

$$Ay(k) = Bu(k-1) + \xi(k)/\Delta \quad (23)$$

where:

- $y(k)$ is the plant output at time k.
- $u(k)$ is the plant input at time k.
- A y B are polynomials in q^{-1}.
- ξ is an ergodic signal that disturbs the system.

In this case the PDE takes the form:

$$1 = E_j \tilde{A} + q^{-j} F_j \quad (24)$$

Where for simplicity S ≡ R ≡ 1.

For this scheme two types of analysis were made. The first of them was to analyse closed loop robustness properties in the face of modelling errors. The second study consisted in determining the closed loop behaviour with GPC for plants with time delay. The main arguments used and results are described next.

4.2 Robustness analysis

By using the analytical solution to the PDE is possible to study how plant modelling errors affect GPC when the prediction horizon grows.
The starting point for this analysis is to establish the asymptotic behaviour of PDE solution errors when plant parameters are known only approximately.
In what follows, super indexes "t" and "e" are used to denote true and estimated quantities, respectively.
Firstly, assume that A^t and A^e are unstable. By using (21), the Z transform of the absolute error in $f_{j,0}$ (given by $\underset{abs}{err} f_{j,0} = f^e_{j,0} - f^t_{j,0}$) is

$$Z\{\underset{abs}{err} f_{j,0}\} = \frac{z}{\Delta} \frac{A^t - A^e}{A^t A^e} \quad (25)$$

From (25) and (21), an expression for the relative error in $f_{j,0}$ (given by $\underset{rel}{err} f_{j,0} = (f^e_{j,0} - f^t_{j,0})/f^t_{j,0}$) is:

$$\underset{rel}{err} f_{j,0} = \frac{Z^{-1}\left\{\frac{z}{\Delta}\frac{1}{A^e}\right\}}{Z^{-1}\left\{\frac{z}{\Delta}\frac{1}{A^t}\right\}} - 1 \quad (26)$$

Recall that the region of convergence for the Z transform is determined by the pole furthest form the origin. Hence, the limit of (26) when j tends to infinity, can be obtained by dividing numerator and denominator terms by the term corresponding to that pole.

By analysing (26) the following result is found:

If after eliminating common factors between A^t and A^e the pole furthest from the origin belongs to A^t, the relative error in $f_{j,0}$ tends to -1; if it belongs to A^e, the limit is $\pm\infty$.

It can also be shown that the prediction obtained from (23) by PDE application has a relative error of

$$\underset{rel}{err}\,\hat{y}_j = (1 + \underset{rel}{err}\,f_{j,0})K_j - 1 \tag{27}$$

with

$$K_j = \frac{\frac{E_j^e}{f_{j,0}^e} B^e \Delta u(k+j-1) + \frac{F_j^e}{f_{j,0}^e} y(k)}{\frac{E_j^t}{f_{j,0}^t} B^t \Delta u(k+j-1) + \frac{F_j^t}{f_{j,0}^t} y(k)} \tag{28}$$

Hence, the asymptotic behaviour of $\underset{rel}{err}\,f_{j,0}$ is inherited by $\underset{rel}{err}\,\hat{y}_j$.

The estimated cost function (with error) can be written in terms of quantities without error, as:

$$J^e(k) = E\left\{\sum_{j=1}^{M} (1 + \underset{rel}{err}\,\hat{y}_j)^2 \left[\hat{y}^t(k+j) - \frac{w(k)}{1+\underset{rel}{err}\,\hat{y}_j}\right]^2 + \lambda \sum_{j=1}^{M} \left[\Delta u(k+j-1)\right]^2\right\} + E\left\{\sum_{j=1}^{M} \xi^2(k+j)\right\} \tag{29}$$

Minimising (29) is equivalent to minimising the true function (given by (22) using true values), with a reference of $w_j = \frac{w(k)}{1+\underset{rel}{err}\,\hat{y}_j}$ and with a relative weighting (between future errors and actuation amplitude) of $\lambda_j = \frac{\lambda}{(1+\underset{rel}{err}\,\hat{y}_j)^2}$. Therefore, if $\underset{rel}{err}\,\hat{y}_{j\to\infty} \to \pm\infty$ (that is so when $f_{j\to\infty,0} \to \pm\infty$), the system follows a zero reference, without keeping the actuation bounded. On the other side, if $\underset{rel}{err}\,\hat{y}_{j\to\infty} \to -1$ (that is so when $f_{j\to\infty,0} \to -1$), the control behaves as if both the reference *w(k)* and the relative weighting λ were infinite. In both cases the control system deteriorates notably.

Therefore for an unstable system and large prediction horizon, it is difficult to get good results for a GPC design based on estimated values, even so when estimation errors are small.

When A^t and A^e are stable, a similar procedure gives the asymptotic behaviour for the relative error in $f_{j,0}$:

$$\underset{rel}{err}\,f_{j\to\infty,o} \to \frac{A^r(1)}{A^e(1)} - 1 \tag{30}$$

By analysing (30) it can be established that for a stable system and large prediction horizon, it is quite possible to get good results for a GPC design based on estimated values.

4.3 Plants with time delay

The closed-loop system can be described by

$$\frac{y(k)}{w(k)} = \frac{KBq^{-1}}{A\Delta P + Lq^{-1}B} \tag{31}$$

where P is a polynomial with the same degree as B and

$$L = \sum_{i=1}^{M} c_i (f_{i,0} + f_{i,1}q^{-1} + f_{i,2}q^{-2}) \tag{32}$$

The K constant and c_i coefficients come from the GPC application. For plants with a time delay: $c_i = 0,\ \forall i \in \{1,2,\ldots,l\}$, where l is the plant time delay. For a second order stable plant with:

$$A(q^{-1}) = (\alpha q^{-1} - 1)(\beta q^{-1} - 1) = a_2 q^{-2} + a_1 q^{-1} + 1 \tag{33}$$

application of (21) gives $f_{j,0}$. Using $i=1,2$ in (14) gives expressions for $f_{j,1}$ and $f_{j,2}$:

$$\begin{aligned} f_{j,0} &= \mu_1 + \mu_2\alpha^{j-1} + \mu_3\beta^{j-1} \\ f_{j,1} &= a_1\mu_1 + a_2(\mu_2\alpha^{j-3} + \mu_3\beta^{j-3}) + \\ &\quad + (a_1 - a_2)(\mu_2\alpha^{j-2} + \mu_3\beta^{i-2}) \\ f_{j,2} &= a_2\mu_1 + a_2(\mu_2\alpha^{j-2} + \mu_3\beta^{j-2}) \end{aligned} \tag{34}$$

where μ_1, μ_2, μ_3 are constants derived from the partial fraction expansion of (21).

Assuming $|\alpha| < 1$, $|\beta| < 1$ and l large enough, some terms in (34) can be neglected, and $L \approx A\mu_1 \sum_{i=l+1}^{M} c_i$, so

$$\frac{y(k)}{w(k)} \approx \frac{Bq^{-1}}{A} \frac{K}{\Delta P + \mu_1 q^{-1} B \sum_{i=l+1}^{M} c_i} \tag{35}$$

In other words, the open loop behaviour tends to be inherited by the closed loop system, as plant delay becomes larger.

Following a similar procedure for plants with poles outside the unit disk, it can be shown that the closed loop system tends to have a factor $1/\Delta$, as plant delay becomes larger.

The time delay necessary for neglected terms in (34) become ε ($0< \varepsilon <1$) times smaller than terms to be kept, is called the critical delay l_o. Values for l_o depend on plant poles locations, and therefore a case by case analysis is necessary.

As an application of this asymptotic analysis, critical delays l_o for second order plants, as functions of open loop denominator characteristics and factor ε, are shown in Table 1. When both poles are on the unit circle, there are no terms to be neglected and the asymptotic analysis does not apply. Table 1 does not include this last case. Table 1 is used as follows: For each pole category and a given factor ε, l_0 is the smallest integer satisfying every corresponding condition.

Using Table 1 with $\varepsilon= 0.001$, that is, neglecting terms on a 1000 to 1 basis, Figure 1 shows predicted critical delays l_o for different plant pole locations. When a_2 is zero, we are dealing with a first order plant, and Figure 1 has been completed using continuity with adjacent points. Critical delays l_o larger than 20, are shown equal to 20, in order to maximise figure clarity.

Figure 1 shows l_o being quite sensitive to plant pole locations for second order systems, displaying valleys in the parameter plane where predicted closed loop behaviour given by (35) is very likely to appear in GPC performance. This may happen even for plant delays as small as 4 sampling periods. On the other side, Figure 1 also shows maximums where much larger delays are allowed, in particular for the well known triangular stability region for second order systems.

The same asymptotic analysis regarding GPC closed loop performance, certainly with more algebraic effort, may be worked out for higher order systems.

TABLE 1 - Restrictions for lo versus plant poles

Pole types	Restriction for stable pole(s)	Restriction for unstable pole(s)
Complex Conjugate	$l_O = \frac{1}{\log\lvert\alpha\rvert}\log\left\lvert\varepsilon\frac{4a_2-a_1^2}{a_1+a_2+1}\right\rvert$	$l_O = \frac{1}{\log\lvert\alpha\rvert}\log\left\lvert\frac{1}{\varepsilon}\frac{4a_2-a_1^2}{a_1+a_2+1}\right\rvert$
Real and distinct	$l_{OA} = \frac{1}{\log\lvert\alpha\rvert}\log\left\lvert\varepsilon\frac{\sqrt{a_1^2-4a_2}}{\beta-1}\right\rvert$ $l_{OB} = \frac{1}{\log\lvert\beta\rvert}\log\left\lvert\varepsilon\frac{\sqrt{a_1^2-4a_2}}{\alpha-1}\right\rvert$	$l_{OA} = \frac{1}{\log\lvert\alpha\rvert}\log\left\lvert\frac{1}{\varepsilon}\frac{\sqrt{a_1^2-4a_2}}{\beta-1}\right\rvert$ $l_{OB} = \frac{1}{\log\lvert\beta\rvert}\log\left\lvert\frac{1}{\varepsilon}\frac{\sqrt{a_1^2-4a_2}}{\alpha-1}\right\rvert$
Real and equal	$l_{OA} = \frac{1}{\log\lvert\alpha\rvert}\log\left\lvert\varepsilon\frac{a_1}{a_1+3}\right\rvert$ l_{OB} = minimum j such that $\lvert(j-3)\alpha^j\rvert < \left\lvert\varepsilon\frac{\alpha}{\alpha-1}\right\rvert$	$l_{OA} = \frac{1}{\log\lvert\alpha\rvert}\log\left\lvert\frac{1}{\varepsilon}\frac{a_1}{a_1+3}\right\rvert$ l_{OB} = minimum j such that $\lvert(j-3)\alpha^j\rvert < \left\lvert\frac{1}{\varepsilon}\frac{\alpha}{\alpha-1}\right\rvert$ $l_{OC} = \varepsilon + 2$ $l_{OD} = \left\lvert\frac{\varepsilon a_2}{\alpha(a_2+\alpha)}\right\rvert + 2$
Real, one at z=1	$l_{OA} = \frac{1}{\log\lvert\alpha\rvert}\log\left\lvert\varepsilon\alpha^2\frac{(1-a_1-a_2)(1-\alpha)-\alpha}{(a_1\alpha+a_2)}\right\rvert$ l_{OB} = minimum j such that $\lvert\varepsilon(j-3)\rvert > \lvert(a_1\alpha+a_2)\alpha^{j-2}\rvert$ $l_{OC} = \frac{1+\varepsilon}{\varepsilon}$ $l_{OD} = \left\lvert\frac{a_2}{\varepsilon a_1}\right\rvert + 2$	$l_{OA} = \frac{1}{\log\lvert\alpha\rvert}\log\left\lvert\frac{\alpha^2(1-a_1-a_2)(1-\alpha)-\alpha}{\varepsilon(a_1\alpha+a_2)}\right\rvert$ l_{OB} = minimum j such that $\lvert j-3\rvert > \lvert\varepsilon(a_1\alpha+a_2)\alpha^{j-2}\rvert$

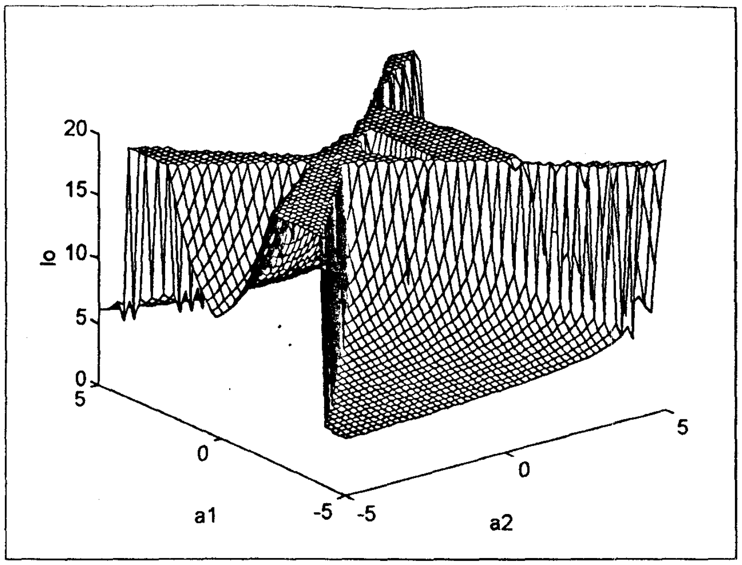

Figure 1: Critical delays l_o for different plant pole locations

CONCLUSIONS

A novel analytical solution to the Polynomial Diophantine Equation (PDE) was proposed. This development is the main result of the paper and it allows to observe clearly the asymptotic behaviour of PDE solutions, constituting a new analysis tool for predictive methods where PDE is widely applied.
As applications of the analytical PDE solution proposed, this tool was used to produce asymptotic results on closed loop performance for the basic GPC method. Specifically, results on robustness in the face of plant modelling errors and on the closed loop performance for plants with time delay were given.
The new approach developed has the advantage of giving explicit solutions to the PDE, but the difficulty of requiring plant pole positions, which are not always available in adaptive control.

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